

COEFFICIENT BOUND ASSOCIATED WITH CERTAIN HANKEL DETERMINANTS AND ZALCMAN CONJECTURE FOR A SUBFAMILY OF MULTIVALENT BOUNDED TURNING FUNCTIONS

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Abstract: In this paper, we introduce certain subfamily of p -valent analytic functions of bounded turning for which we estimate best possible upper bound to certain generalised second Hankel determinant, the Zalcman conjecture and an upper bound to the third, fourth Hankel determinants. Further, we investigate an upper bound for third and fourth Hankel determinants with respect to two-fold and three-fold symmetric functions for the same class. The practical tools applied in the derivation of our main results are the coefficient inequalities of the Carathéodory class \mathcal{P} .

Keywords: p -valent holomorphic bounded turning function, upper bound, Hankel determinant, univalent function, Carathéodory function, hypergeometric function

1. Introduction

Let \mathcal{A}_p , with $p \in \mathbb{N} = \{1, 2, 3, \dots\}$ represent the family of all p -valent analytic mappings f of the form

$$f(z) = z^p \sum_{n=0}^{\infty} a_{p+n} z^n, \quad a_p := 1, \quad z \in \mathbb{D}, \quad (1.1)$$

in the open unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ and $\mathcal{A}_1 = \mathcal{A}$, be the subfamily of all analytic normalized functions f of the form

$$f(z) = \sum_{n=1}^{\infty} a_n z^n, \quad a_1 := 1, \quad z \in \mathbb{D}. \quad (1.2)$$

By S , we denote the subfamily of \mathcal{A} , consisting of all univalent functions. Pommerenke [19] characterized the r^{th} -Hankel determinant of degree t , for f in (1.2),